$X \sim f_X(x)$

$Y \sim f_Y(y)$

$(X, Y) \sim f_{XY}(x, y)$

Distribution of $X+Y$?

$$P(X+Y \le z) = \int P(X+Y \le z \mid X=x) f_X(x)\, dx$$

$$= \int P(Y \le z-x \mid X=x) f_X(x)\, dx$$

If X and Y are independent,

$$= \int P(Y \le z-x) f_X(x)\, dx$$

$$\Rightarrow f_{X+Y}(z) = \frac{d}{dz} P(X+Y \le z)$$

$$= \frac{d}{dz} \int \overbrace{P(Y \le z-x)}^{F_Y(z-x)} f_X(x)\, dx$$

$\{ f_Y(u) = \frac{d}{du} F_Y(u) \}$

$$= \int f_Y(z-x) f_X(x)\, dx$$

$$= [f_X * f_Y](z)$$

convolution →